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Mathematical Modeling and Simulation of Quadcopter-UAV Using PID Controller

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Abstract: *- There has been a lot of experimental, analytical and computational work carried out in field of robotic systems for quadcopter-UAV in the last two decades. In the recent times, focus has shifted to analytical solutions particularly to ensure balancing and accurate trajectory control in quadcopters with a need for better accuracy. The developed models are simulated and simplified and even improvised using an appropriate software tool. MATLAB is very popularly used for analysis and validation of the models developed. The modeling of such systems was done based on first principles of kinematics and dynamics. Specifically, a large tracking error is produced in the quadcopter system due to the external disturbances, which poses problems in controller design. Hence, in this work, firstly kinematics is framed from a different perspective. This is followed by an effective use of Jacobi method to derive equations with respect to the body frame of reference and inertial frame of reference. Solutions have been obtained for important parameters such as linear acceleration, angular acceleration and torque.*

Keywords: - *Quadcopter, Control, Mathematical modeling, Newton-Euler method, Inertia frame, Body frame.*

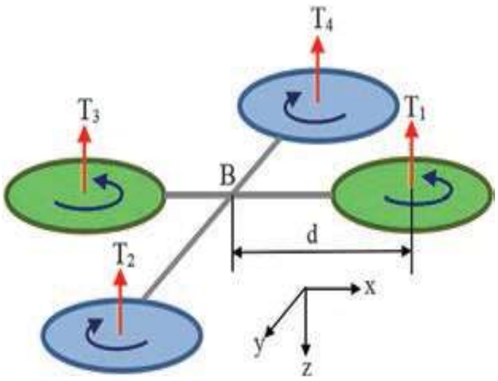
I. Introduction

A quadcopter is unmanned aerial vehicle with four-rotor to control its motion in six degrees of freedom (6DOF). The pairs of rotors rotate in clockwise - anticlockwise direction [1]. For rolling and Pitching, Quadcopter tilts toward the direction of the slow spinning motor. Linear Motion is achieved by dividing the thrust into two directions which are done by Roll and pitch angles. It is very difficult to stabilize a quadcopter by human control. Hence, a sophisticated controller is used for a balanced flight of quadcopter [2]. Most of the practicing engineers and researchers have been using quadcopter for an incredible growth in applications and its simple mechanical design. The four propulsors affects position and attitude of quadcopter in space, hence there are four controllable variables are used in quadcopter for balancing. With change in torque with respect to the one of the axes quadcopter inclination is achieved [3]. A challenge of PID control method is controlling the disturbance that produces large error in the systems and also the time-varying response of systems in terms of settling time. Steady-state response and overshoots are approximated so that it presents an ability to withstand rigorous testing conditions. Hence, in this work, firstly kinematics is framed from a different perspective. This is followed by an effective use of Jacobi method to derive equations with respect to the body frame of reference and inertial frame of reference. Solutions have been obtained for important parameters such as linear acceleration, angular acceleration and torque. The developed mathematical framework is implemented in MATLAB. A major feature of developed equations is that solution is obtained with small discrete steps in MATLAB.

A mathematical model derivation, coordinate systems are defined and explained. By using those coordinate systems, relations between parameters defined in the earth coordinate system and in the body coordinate system are defined.

II. Literature Survey

A quadcopter has four motors mounted at the ends of cross arm which are labelled as 1 through 4. The motor 1 & 3 rotate in counter clockwise whereas motor 2 & 4 rotates in clockwise direction. The center of the body-fixed frame B is the center of mass and the origin, which is attached to the quadcopter. Rotor 1, 2, 3 and 4 produces upward thrust T1, T2, T3 and T4 respectively, and d is the length between the center of mass and center of rotor shown in fig.1 [4]. A mathematical model of a quadrotor dynamics had derived, using Newton's and Euler's laws. The Simulink 3D Animation toolbox is used to observe the behaviour of the quadrotor in computer-generated simulation, also the quadcopter behavior is observed for different speed of rotors for the roll, pitch and yaw angles with respect to time [3, 5]. Fig. 1 Quadcopter notation showing the four motors. The different controllers are designed in order to maintain stability of quadcopter for a robust control law.



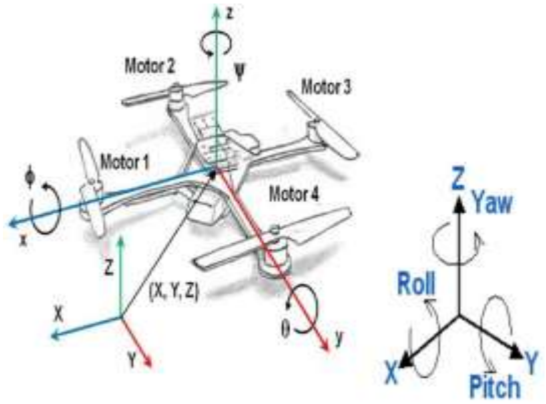


Fig. 1 Quadcopter notation of four motors [4]. **Fig. 2** Coordinate systems and Roll, Pitch, Yaw angles [9]

To implement this control, there are many different algorithms has been designed, which sometimes uses model-free controllers employing fuzzy logic algorithms, neural networks and genetic algorithms, proportional integral derivative (PID) controllers, Linear Quadratic Regulator (LQR) for controlling mobile robots [6.7,4]. The modern mathematical formulation techniques are used to describe the classical mechanics theories on rigid body motion. This modern mathematical formulation is made with help of the differential manifold and differential Topology method. A mathematical model has been developed for trajectory control by using kinematics and dynamics of a quadcopter and algorithms [8]. A PD controller is used for control theory & flight control algorithms for stabilization of the quadcopter attitude and newly method is developed to control the trajectory of the quadcopter. The PD controller was integrated into a new method for better response to disturbances in the flight conditions of the quadcopter [7, 8].

Newton-Euler and Euler-Lagrange equations are used to derive differential equations for a mathematical model of a quadrotor dynamics. The Mathematical model derived for position & altitude control of quadcopter provides efficient stability at desired altitude and attitude with use of PD controller [5, 7]. The dynamic inversion with zero dynamics stabilization, based on static feedback linearization obtaining a partial linearization of the mathematical model, and the exact linearization and non-interacting control via dynamic feedback, based on dynamic feedback linearization obtaining a total linearization of the mathematical model these are the two feedback linearization control schemes has been derived [5]. We have used rotational matrix equations from mentioned paper to find angular velocity in all axis of the system [1]. Mathematical model is developed to sustain the effects of air drag and possible disturbance [3]. 6 DOF state space model derived from basic Newtonian equations are also used for finding results in all DOF [10]. Linearization of state space model which had conducted in the mentioned paper also refers in our formulations [8]. To stabilize the altitude performance, PID controller was implemented [11]. Euler and Lagrangian mechanics for calculating energy equations used in the mentioned paper are also studied to formulate our final equations [9]. If two opposite rotors rotate at the equal speed in opposite directions the total yaw force will be zero. As the speed of opposite pairs changes, the pitch & roll of quadcopter also changes [4, 12]. The mathematical model is develop for comparing experimental identification and the computation of model parameters like thrust coefficient, drag coefficient, inertia matrix, translational and rotational drag coefficients [1]. The main feature of designed quadcopter model is a fully-controlled system which can track any arbitrary trajectory with controlled pitch and roll angle, and motion with desired orientation [12]. A dynamic model of a quadcopter developed for comparing Classic and cascade PID controller to demonstrate the effectiveness of the designed controller using a robust cascade PID control algorithm [2]. For simulation purpose the developed mathematical model is programmed within the MATLAB environment for different engagement scenarios and different sources of uncertainties. The performance of designed model is observed in MATLAB or other similar software using different control techniques like PD, PID, LQR, and H1 controller [2, 11, 13, 14].A unique mathematical model of quadcopter UAV in 6 DOF has been derived from basic Newtonian equations for representing a set of input, output and state variables related to first-order differential equations which is easy to understand dynamics of quadcopter for the readers at a root level [10].

2.1 Strategies Studied From Literature Survey

We have used rotational matrix equations from mentioned paper to find angular velocity in all axis of the system [8]. Euler and Lagrangian mechanics for calculating energy equations used in the mentioned paper are also studied to formulate our final equations [3]. A mathematical model is developed to sustain the effects of air drag and possible disturbance [1]. 6 DOF state space model derived from basic Newtonian equations are also used for finding results in all DOF [9]. If the speed of opposite pairs changes the pitch & roll of quadcopter also changes [2]. For developing robust PID controller algorithms for our model we referred mentioned paper controller formulations [10]. The main feature of designed quadcopter model is a fully-controlled system which can track any arbitrary trajectory with controlled pitch and roll angle, and motion with desired orientation [5]. Linearization of state space model which had conducted in the mentioned paper also refers in our formulations [6]. To stabilize the altitude performance PID controller was implemented [11]. For simulation purpose the developed mathematical model is programmed within the MATLAB environment for different engagement scenarios and different sources of uncertainties. The performance of designed model is observed in MATLAB or other similar software using different control techniques like PD, PID, LQR, and H1 controller [10, 11].

III. Mathematical Modeling

The mathematical model is developed using Newton-Euler method and Euler-Lagrangian method with following steps.

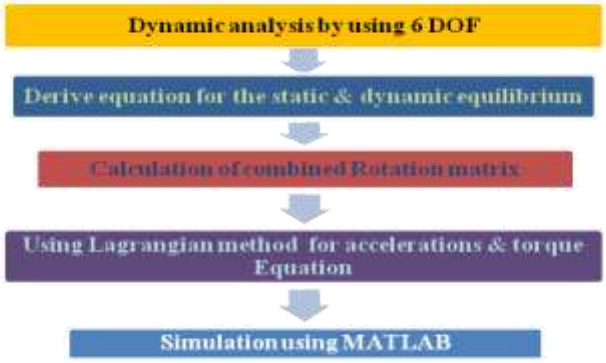


Fig. 3 Methodology

The quadcopter structure is presented in Fig. 4 including the corresponding angular velocities, torques and forces created by the four rotors. The quadcopter dynamics is modeled with respect to reference system in which the inertial frame related to body frame [9].

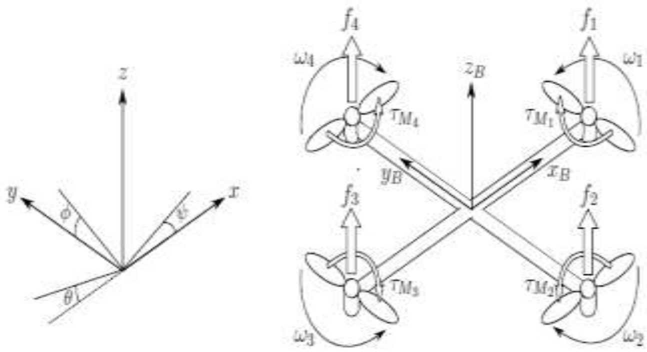


Fig. 4 The inertial and body frame of a quadcopter [7].

3.1 Direction-Cosign matrix

$\varepsilon \rightarrow$ Absolute linear position of Inertial frame

$\eta \rightarrow$ Attitude i. e. Angular position of Inertial frame

$\delta \rightarrow$ Linear & *angular position vectors*

$\emptyset$ = Rotational movement along X − axis

θ = Pitch movement around Y − axis

φ = Yaw movement around Z − axis

Rotation Matrix $[R] = [Z][Y][X]$ …………………………………………………………... (1)

The rotation matrix from the body frame to the inertial frame

$$[\mathrm{R}] = \begin{bmatrix} \cos\theta\cos\varphi & \sin\theta\sin\emptyset\cos\varphi - \sin\varphi\cos\emptyset & \sin\theta\cos\emptyset\cos\varphi + \sin\emptyset\sin\varphi \\ \sin\varphi\cos\theta & \sin\emptyset\sin\theta\sin\varphi & \sin\theta\sin\varphi\cos\emptyset - \sin\emptyset\sin\varphi \\ -\sin\theta & \sin\emptyset\cos\theta & \cos\theta\cos\emptyset \end{bmatrix}$$

$$\therefore [\mathrm{R}]^T = \begin{bmatrix} \cos\theta\cos\varphi & \sin\varphi\cos\theta & -\sin\theta \\ \sin\theta\sin\emptyset\cos\varphi - \sin\varphi\cos\emptyset & \sin\emptyset\sin\theta\sin\varphi & \sin\emptyset\cos\theta \\ \sin\theta\cos\emptyset\cos\varphi + \sin\emptyset\sin\varphi & \sin\theta\sin\varphi\cos\emptyset - \sin\emptyset\sin\varphi & \cos\theta\cos\emptyset \end{bmatrix} \ldots\ldots\ldots\ldots (2)$$

$$[v] = [\mathrm{R}]^T[\omega_i]$$

$$\begin{bmatrix} v_\phi \\ v_\theta \\ v_\psi \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\emptyset & \sin\emptyset\cos\theta \\ 0 & -\sin\emptyset & \cos\theta\cos\emptyset \end{bmatrix} \begin{bmatrix} \dot{\emptyset} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix}$$

$$\therefore v = \omega_\eta \dot{\eta}$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\emptyset & \sin\emptyset\cos\theta \\ 0 & -\sin\emptyset & \cos\theta\cos\emptyset \end{bmatrix} \begin{bmatrix} \dot{\emptyset} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Quad-copter is symmetrical in structure hence; the inertia matrix is diagonal matrix I in which Ixx = Iyy.

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

There are 4 rotors hence force fi with angular velocity in the direction of rotor axis creates torque

Now

k_L = Lift constant

Combine forces of rotor create thrust T in Z-axis;

$$T = \sum_{i=1}^{4} \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$\tau_B = \begin{bmatrix} \tau_\emptyset \\ \tau_\theta \\ \tau_\varphi \end{bmatrix} = \begin{bmatrix} l k_L({\omega_4}^2 - {\omega_2}^2) \\ l k_L({\omega_3}^2 - {\omega_1}^2) \\ \sum_{i=1}^{4} \tau_{mi} \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

3.2 Dynamic analysis

Using the Lagrangian method

$$L = E_{transl.} + E_{rotati.} - E_{potent.}$$

External forces & torques

$$\begin{pmatrix} f \\ \tau \end{pmatrix} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\delta}}\right) - \frac{\partial L}{\partial \delta} \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

$$f = T \begin{bmatrix} \sin\theta\cos\emptyset\cos\varphi + \sin\emptyset\sin\varphi \\ \sin\theta\sin\varphi\cos\emptyset - \sin\emptyset\sin\varphi \\ \cos\theta\cos\emptyset \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

The linear Lagrangian equation is given by;

$$f = RT_B = m\ddot{x} + kx$$

This is for X direction. For all degree of freedom;

$$RT_B = m\ddot{\varepsilon} + k\varepsilon$$

$$\ddot{\varepsilon} = \frac{RT_B}{m} - \frac{k\varepsilon}{m}$$

$$\ddot{\varepsilon} = \frac{T_B}{m}\begin{bmatrix}\sin\theta\cos\emptyset\cos\varphi + \sin\emptyset\sin\varphi \\ \sin\theta\sin\varphi\cos\emptyset - \sin\emptyset\sin\varphi \\ \cos\theta\cos\emptyset\end{bmatrix} - \frac{k}{m}\begin{bmatrix}x\\y\\z\end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Considering angular motion;

$[\omega_\eta]^{-1}$ = Angular velocity transformation matrix for Inertial frame to body frame

$$[\omega_\eta][\omega_\eta]^{-1} = [\,\mathrm{I}\,]$$

$$[\omega_\eta]^{-1} = \begin{bmatrix}1 & \sin\emptyset\tan\theta & \cos\emptyset\tan\theta \\ 0 & \cos\emptyset & -\sin\emptyset \\ 0 & \sin\emptyset/\cos\theta & \cos\emptyset/\cos\theta\end{bmatrix}$$

Now angular acceleration of the inertial frame is time derivative of the transformation matrix from body frame.

$$\ddot{\eta} = \frac{d({\omega_\eta}^{-1}v)}{dt}$$

$$\therefore \ddot{\eta} = \begin{bmatrix}1 & \sin\emptyset\tan\theta & \cos\emptyset\tan\theta \\ 0 & \cos\emptyset & -\sin\emptyset \\ 0 & \sin\emptyset/\cos\theta & \cos\emptyset/\cos\theta\end{bmatrix}\begin{bmatrix}p\\q\\r\end{bmatrix}$$

$$\therefore \ddot{\eta} = \begin{bmatrix}p + q(\sin\emptyset\tan\theta) + r(\cos\emptyset\tan\theta) \\ q(\cos\emptyset) - r(\sin\emptyset) \\ q(\sin\emptyset/\cos\theta) + r(\cos\emptyset/\cos\theta)\end{bmatrix}$$

$$\therefore \ddot{\eta} = \begin{bmatrix}0 & \tan\theta\cos\emptyset\,\dot{\emptyset} + \sin\emptyset\dfrac{\theta}{cos\theta^2} & -\dot{\emptyset}\sin\emptyset\tan\theta + \dfrac{\cos\emptyset\dot{\theta}}{cos\theta^2} \\ 0 & -\sin\emptyset\dot{\emptyset} & -\cos\emptyset\dot{\emptyset} \\ 0 & \dfrac{\cos\emptyset\dot{\emptyset}}{\cos\theta} + \dfrac{\sin\emptyset\tan\theta\,\dot{\theta}}{\cos\theta} & -\dfrac{\sin\emptyset\dot{\emptyset}}{\cos\theta} + \dfrac{\tan\theta\cos\emptyset\,\dot{\theta}}{\cos\theta}\end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

3.3 Angular accelerations Equations

$$\therefore \ddot{\eta}_1 = \ddot{\emptyset}\sin\emptyset\tan\theta + \ddot{\varphi}\sin\theta\,(2 - \sin\emptyset\tan\theta) + \dot{\emptyset}\dot{\varphi}(-\sin\emptyset\sin\theta - 2\sin\theta\tan\theta + \sec\theta)$$
$$\therefore \ddot{\eta}_2 = \ddot{\theta} + \dot{\emptyset}\dot{\varphi}\cos\theta$$
$$\therefore \ddot{\eta}_3 = 2\ddot{\varphi} - \dot{\varphi}\dot{\theta}\tan\theta$$

3.4 Torque Equations

$$\tau_1 = \mathrm{I}_{xx}[\ddot{\emptyset}\sin\emptyset\tan\theta - \ddot{\varphi}\sin\theta\tan\theta\sin\emptyset + \dot{\emptyset}\dot{\varphi}(\sec\theta - \sin\theta\sin\emptyset)]$$
$$\tau_2 = \ddot{\theta}(\mathrm{I}_{yy}\cos\emptyset^2 + \mathrm{I}_{zz}\sin\emptyset^2) + [\dot{\emptyset}\dot{\varphi}\cos\theta\,(\mathrm{I}_{yy}\cos\emptyset^2 + \mathrm{I}_{zz}\sin\emptyset^2)] + 2\ddot{\varphi}\cos\theta\sin\emptyset\cos\emptyset\,(\mathrm{I}_{yy} - \mathrm{I}_{zz}) + \dot{\varphi}\dot{\theta}\sin\theta\sin\emptyset\cos\emptyset\,(\mathrm{I}_{zz} - \mathrm{I}_{yy})$$

$$\tau_3 = I_{xx}\left[-\ddot{\emptyset}\tan\theta\sin\emptyset\sin\theta + \ddot{\varphi}\sin\emptyset\tan\theta\sin\theta^2 + \dot{\emptyset}\dot{\varphi}(\sin\theta^2\sin\emptyset - \tan\theta)\right] + \ddot{\theta}\cos\theta\sin\emptyset\cos\emptyset\left(I_{yy} - I_{zz}\right) + \dot{\emptyset}\dot{\varphi}\sin\emptyset\cos\emptyset\cos\theta^2\left(I_{yy} - I_{zz}\right) + 2\ddot{\varphi}\cos\theta^2\left(I_{yy}\sin\emptyset^2 + I_{zz}\cos\emptyset^2\right) - \dot{\emptyset}\dot{\varphi}\cos\theta\sin\theta\left(I_{yy}\sin\emptyset^2 + I_{zz}\cos\emptyset^2\right)$$

3.5 Acceleration Equations

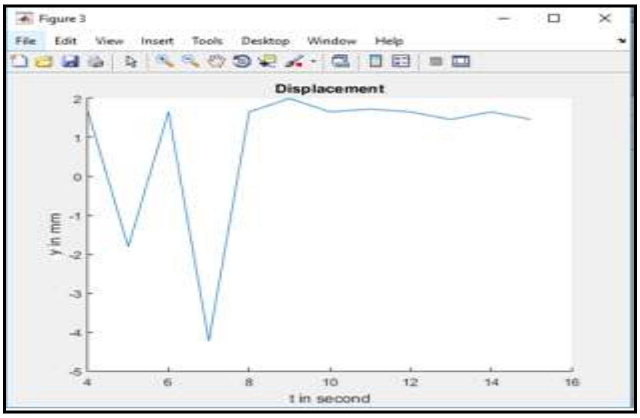
$$\ddot{\varepsilon}_1 = \frac{T}{m}(\sin\theta\cos\emptyset\cos\varphi + \sin\emptyset\sin\varphi) + \frac{k}{m}(x)$$

$$\ddot{\varepsilon}_2 = \frac{T}{m}(\sin\theta\cos\emptyset\sin\varphi - \sin\emptyset\cos\varphi) + \frac{k}{m}(y)$$

$$\ddot{\varepsilon}_3 = \frac{T}{m}(\cos\theta\cos\emptyset) + \frac{k}{m}(z)$$

IV. Result Discussion

Graph 4.1

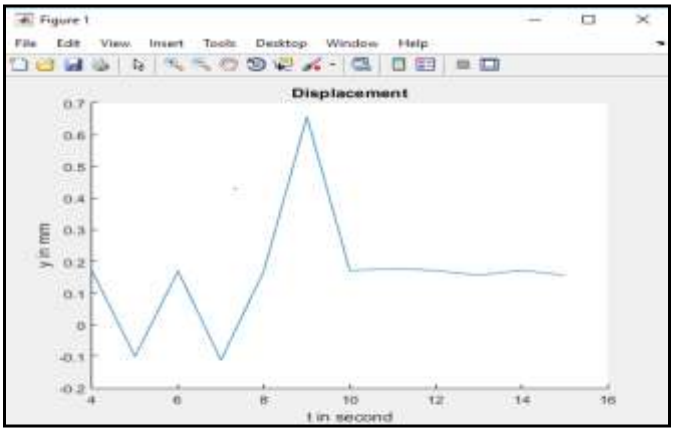


Graph 4.1 – Displacement (mm) Vs Time (sec)

Kp=0.4, Kd=0.5, Ki=0.2 ω1= 50 rad/sec; ω2= 150 rad/sec; ω3= 150 rad/sec; ω4= 200 rad/sec

Effect of speed closeness is dominant as observed from the graph above. Thus it can be concluded that sensitivity analysis is well captured by developed mathematical framework.

Graph 4.2

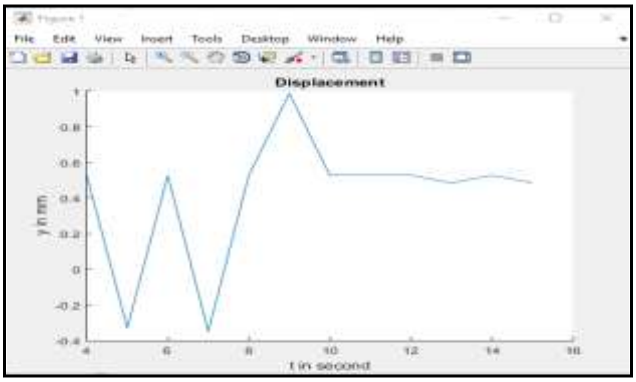


Graph 4.2 – Displacement (mm) Vs Time (sec)

Kp=0.4, Kd=0.5, Ki=0.2 ω1= 50 rad/sec; ω2= 100 rad/sec; ω3= 150 rad/sec; ω4= 200 rad/sec

For this combination, it seems that displacement in the context of Y-direction flutters initially nearly sinusoidal within a period of 8 seconds. Maximum negative found to be 0.1mm while positive goes to 0.7mm. Distance has got a major role in controlling response. After this, the system is observing to be stable. Response time taken is 10seconds. Unsymmetrical nature of curves observed through. Overall response characteristics are found to be smooth and no sudden jerks are observed. Thus equations, discretizations, and control found to be perfect.

Graph 4.3

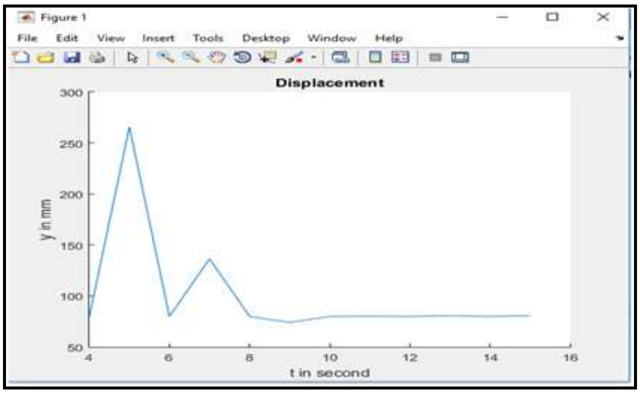


Graph 4.3 – Displacement (mm) Vs Time (sec)

Kp=0.4, Kd=0.6, Ki=0.2 ω1= 50 rad/sec; ω2= 150 rad/sec; ω3= 180 rad/sec; ω4= 200 rad/sec.

For this combination, results are similar to the previous case but again it seems that displacement in the context of Y-direction flutters initially nearly sinusoidal within a period of 8 seconds. Maximum negative found to be 0.35mm while positive goes to 1mm. Kd has got significant an effect on the spectrum. After this, the system is observing to be stable. Response time taken is close to 10 seconds. Unsymmetrical nature of curves observed though. Overall response characteristics are found to be smooth and no sudden jerks are observed. Thus equations, discretizations, and control found to be perfect.

Graph 4.4



Graph 4.4 – Displacement (mm) Vs Time (sec)

Kp=0.4, Kd=0.6, Ki=0.2 ω1= 150 rad/sec; ω2= 200 rad/sec; ω3= 150 rad/sec; ω4= 200 rad/sec

Distance has got a major role in controlling response. Maximum Y goes to be around 270 mm. Response lasts at 10 seconds to be unstable thereafter the response becoming stable. Here also response is sinusoidal which is appropriate as the system is hinting to be non vulnerable.

V. Conclusion & Future Scope

In this work, a mathematical model of a quadcopter is derived for angular velocity, acceleration, and torque for all degree of freedom with optimized formulations. The analysis concludes that the quadcopter initially is in a disturbed mode but after gaining more frequency it stabilizes. As the flight time increases the stability becomes constant. The Newton-Euler and Euler-Lagrange method used to design a model to stabilize the quadcopter. A PID controller is used to balances the quadcopter and stable it at high frequency. Formulation and the design of advanced controller which can definitely give more safety to quadcopter performance at the same time other ways to improve system stability can also be investigated. The system must be developed by applying designed techniques and calibrated first. Further the design of the model to determine the remaining terms. New controller parameters should be practically attempted. The strategy should be put in place for specific underlying applications. Simulations based performance should be quantified and compared with actual performance.

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